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FUZZY COMPREHENSIVE EVALUATION (FCE) IN MILITARY DECISION SUPPORT PROCESSES

by

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December 2013

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13. ABSTRACT (maximum 200 words)

The United States has a tradition of military analysis using a federated or combined suite of models. However, these are not the only methods of modeling military problems. We consider the application and implications of foreign modeling approaches. The particular alternate technique we focus on is fuzzy comprehensive evaluation (FCE). FCE makes use of fuzzy mathematics, alone and in partnership with Analytic Hierarchy Process (AHP) models, to inform strategic and operational decisions. It is designed to aid leaders in capturing the complicated and sometimes "fuzzy" nature of multi-criteria decision problems through human knowledge and evaluations. These subjective inputs present criticisms regarding FCE solutions. FCE results are only as valid as the consistency of the subject matter expert's opinions. Therefore, this thesis analyzes the FCE approach through a case study and evaluates the implications of FCE results when there is high variance in expert opinions.

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FUZZY COMPREHENSIVE EVALUATION (FCE) IN MILITARY DECISION SUPPORT PROCESSES

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ABSTRACT

The United States has a tradition of military analysis using a federated or combined suite of models. However, these are not the only methods of modeling military problems. We consider the application and implications of foreign modeling approaches. The particular alternate technique we focus on is fuzzy comprehensive evaluation (FCE). FCE makes use of fuzzy mathematics, alone and in partnership with Analytic Hierarchy Process (AHP) models, to inform strategic and operational decisions. It is designed to aid leaders in capturing the complicated and sometimes "fuzzy" nature of multi-criteria decision problems through human knowledge and evaluations. These subjective inputs present criticisms regarding FCE solutions. FCE results are only as valid as the consistency of the subject matter expert's opinions. Therefore, this thesis analyzes the FCE approach through a case study and evaluates the implications of FCE results when there is high variance in expert opinions.

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LIST OF ACRONYMS AND ABBREVIATIONS

AHP	analytic hierarchy process
AO	area of operation
AOR	area of responsibility
CI	consistency index
CLF	combat logistic force
CLS-5000	Combat Logistic Ship (5000 tons)
CR	consistency ratio
COA	course of action
FCE	fuzzy comprehensive evaluation
JCA	joint campaign analysis
JHSV	Joint High-Speed Vessel
LCS	Littoral Combat Ship
MCDM	multiple criteria decision-making
MLP	Mobile Landing Platform
MILP	mixed integer linear program
MOT	means of transportation
REB	repositionable expeditionary base
R2B	robustness to bias
SAG	surface action group
SCS	South China Sea or S. China Sea
ST	short tons
T-AKE	USNS Lewis and Clark class dry cargo/ammunition ship

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EXECUTIVE SUMMARY

The United States has a tradition of military analysis using a federated or combined suite of models. These are not the only methods of modeling military problems. We consider the application and implications of foreign modeling approaches. The particular alternate technique we focus on is fuzzy comprehensive evaluation (FCE). FCE makes use of fuzzy mathematics, alone and in partnership with the analytic hierarchy process (AHP), to inform strategic and operational decisions. In 1965, Lotfi A. Zadeh introduced fuzzy mathematics as a way to capture uncertainty, particularly when assessing decisions with qualitative factors. Fuzzy mathematics has not gained widespread use in the United States because other methods for dealing with uncertainty are preferred. Additionally, the AHP's purpose is to organize and formulate weights for multiple criteria of a decision process. This thesis accomplishes three research goals:

1. It critiques the application of fuzzy math and AHP in FCE.
2. It finds differences between the FCE approach and current U.S. approaches when applied to military decision support processes.
3. It evaluates the implications of FCE results when there exists a high variance in expert opinions.

We examine the reasons why the results of optimization and FCE diverge. This proves useful for discovering weaknesses in the FCE results, as well as understanding places where there may be strategic miscalculations.

We apply FCE to a military logistics resupply scenario and compare the results with a linear optimization model. The scenario presents three alternative means of transportation (MOT): the Independence variant Littoral Combat Ship (LCS-2), Joint High-Speed Vessel (JHSV), and a modified JHSV, Combat Logistic Ship (5000 tons) (CLS-5000), to supply the wartime demands of combat ships in theatre. The goal is to meet demands at the lowest operating cost of the MOTs. Although combinations of MOTs are possible solutions for the optimization program, the mixed integer linear program (MILP) selected nine CLS-5000s. The MILP was also able to rank the three alternatives from the lowest cost option to highest as follows: 1) CLS-5000 2) JHSV 3) LCS-2. FCE did not contain the ability to inform users of "how many" alternative ships

were required to fulfill demand for this case study. Chapter III details these findings. Furthermore, we develop a simulated FCE study that expects to match the optimization results, and study how potential variation in expert opinion could lead to different or incorrect rankings.

In Chapter III's case study, we found a few criticisms for the FCE modeling technique. The main criticism is in its employment of human knowledge and evaluations for inputs. This method poses risks for introduction of bias and inaccurate results if not handled properly. Generally, the process to ensure the accuracy of expert judgments is that all judgments must be consistent. However, if expert judgments are truly incorrect but pass the consistency check, there is no real way of uncovering this discrepancy. The imprecision of results is also troublesome. FCE can handle the modeling of a few SMEs (subject matter experts). Although a realistic condition, this practice presents issues in precision due to the small sample size of expert input. Additionally, FCE only considers the mode of results as the solution and disregards the distribution of scores. For example, a fuzzy assessment with 52% of an alternative's membership in the "fair" category and its other 48% in the "poor," reflects only an alternative performing in the "fair" category. The almost equal membership of an alternative in the "poor" category is ignored. This reproachful handling of results is combined with contradicting techniques to handle alternatives within the same category. Unfortunately there is no guidance for FCE outcomes when one method of breaking ties presents conflicting results compared to another.

Amidst these pitfalls, FCE presents an alternative decision-making approach that can handle smaller samples of data. This characteristic can prove beneficial when limited data requires analysis. Ballistic missile performance statistics are a prime example. Generally, sample experimentation of this kind is limited due to cost, resource, and policies of the manufacturer and consumer. Instead of waiting for limited or unattainable data, FCE acquires information from an alternative readily available source – experts. Furthermore, with enough experts, FCE's approach to multi-criteria decision problems with subjective judgments can provide quantifiably useful and precise results as long as its pitfalls are considered.

Evidently, FCE results are as good as its subject matter expert inputs. Thus, we created and employed the R2B (robustness to bias) tester in Chapter IV to provide a generally applied method for testing the robustness of expert opinions. R2B tests high variance in subjective inputs (due to the bias in expert opinions) and evaluates the robustness in the subsequent FCE results. This thesis studied simulation replications for sequences of experts with the presence of one adversarial expert who provided intentional bias in each sequence. All simulation iterations involved complete execution of the FCE process. The selection scores for sequences of experts for each alternative were recorded. After the scores were tallied, they were compared to known "true" rankings of the alternatives. In the end, R2B confirmed that the same alternative would still be chosen even with added simulated variability and adversarial inputs. It also highlighted the ratio of good experts required for cogent outcomes when one adversarial expert was present. Moreover, even though results are specific and applicable to the scenario presented to R2B, this method can be applied to any set of expert opinions to estimate the potential effect of bias on FCE results.

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I. INTRODUCTION

"Si vis pacem, para bellum—if you want peace, prepare for war."

—Publius Flavius Vegetius Renatus

A. BACKGROUND

The task of preparing for armed conflict is extremely difficult and complex. This is evident in the face of belligerents who possess various means and ways of waging war. The number of contingency plans countering each possible scenario against all likely adversaries would be endless, the question of how to prepare for such events remains. Perhaps Sun Tzu's teachings can provide enlightenment to this predicament. Sun Tzu (1963) imparted a famous proverb to his generals, "Know the enemy, know yourself; in a hundred battles you will never be in peril" (p. 84). His instructions to the warfighter were meant to impart the role of intelligence and assessment in framing strategies towards winning conflicts. The knowledge gained from understanding an opponent's capabilities can lead to capitulation of the enemy through attacking these strategic sources of strength (i.e., command centers, forces, economy, leadership, etc.). Thus, possessing superior intelligence leads to advanced preparation.

Even though two opponents may view the same problem differently, the one who most understands their opponent's view gains the upper hand. Thus, this understanding poses risks when individuals incorrectly assess their opponent's intentions and opportunities when an enemy's capabilities are correctly anticipated. For example, nuclear conflict during the Cold War led to stability because both the U.S. and Russia correctly assessed the costs of going to war as prohibitively high. This is in stark contrast to the situation during Operation Iraqi Freedom, where the U.S. underestimated the costs of conflict with Iraq and the Iraqis incorrectly assessed it to be high. This miscalculation was disastrous for the Iraqis, but damaging for both sides (Haass, 2009).

Comprehending the mathematical approaches used by potential adversaries helps decision-makers understand where risk miscalculations (by either side) may take place.

This provides leadership with potential "inside knowledge" as to how an adversary makes decisions and prepares their contingency plans. Most importantly this knowledge highlights where likely missteps may take place, both to the advantage and disadvantage of a belligerent.

B. METHODOLOGY

The United States has a tradition of military analysis using a federated suite of models. These are not the only methods of modeling military problems. We consider the application and implications of foreign modeling approaches. The particular alternate technique we focus on is "Fuzzy Comprehensive Evaluation" (FCE).

FCE makes use of fuzzy mathematics, alone and in partnership with the Analytic Hierarchy Process (AHP), to inform strategic and operational decisions. In 1965, Lotfi A. Zadeh introduced fuzzy mathematics as a way to capture uncertainty, particularly when assessing decisions with qualitative factors. Fuzzy mathematics has not gained widespread use in the United States, because other methods for dealing with uncertainty are preferred. AHP works by organizing all aspects or factors of a decision problem and providing a perspective of importance of the components of the decision. Fuzzy math and the AHP collaborate to form FCE, a decision-making tool used to evaluate competing decision alternatives or courses of action influenced by multiple factors. Eventually, the FCE determines which alternative is 'best' at meeting the main objective or decision question. Chapter II explains the FCE methodology.

FCE methods are foreign. Their applications have been found in Chinese research, industry, systems engineering, as well as military applications (Li, Ma, & Liu, 2004). Lotfi A. Zadeh, the creator of fuzzy logic, noted in a 1994 interview with *Azerbaijan International* how the support for fuzzy logic in China is beginning to increase since the loss of funding after events of Tiananmen Square and is "likely to grow substantially" (Blair, 1994, p. 47). Applications in evaluating naval tactical missile systems, attack helicopters, and modeling of U.S. overseas logistics operations are among the few studied by Chinese analysts (Cheng, 1996; Cheng, Yang, & Hwang, 1999; Mu, Guo, Niu, & Jia, 2013). During the interview, Professor Zadeh also highlighted substantial Japanese work in consumer products (i.e., appliances and electronics) using fuzzy logic. The Japanese are the top practitioners of fuzzy-logic based systems and products since the 1980s, according to Dr. Zadeh. With such success of several fuzzy logic based products in the east, a revival in using fuzzy methods began in the U.S. by the early 90s (Blair, 1994). As a result of the revival, the U.S. has considered applying fuzzy methods to military tracking systems (Smith, 1995) and neural networks for anomaly detection in military aircraft (Brotherton & Johnson, 2001). Today, as reported by the international journal *Advances in Fuzzy Systems* (2013), as of March 4, 2013, the U.S. has 16,898 patent applications and patents issued related to fuzzy logic. This is in stark contrast to the 7,149 patent applications and patents issued to fuzzy logic related work in Japan (Singh et al., 2013). But even though fuzzy analysis has gained greater acceptance in the U.S., the academic community still heavily scrutinizes this approach.

C. OBJECTIVE

The goal of this thesis is to introduce the FCE methodology and research and analyze its vulnerabilities. We conduct an analysis of FCE vulnerabilities in a combat logistics resupply scenario and in a study of intentionally biased inputs. The goal is to meet demands at the lowest operating cost of the MOTs. This thesis accomplishes three research goals:

1. It critiques the application of fuzzy math and AHP in FCE.
2. It finds differences between the FCE approach and current U.S. approaches when applied to military decision support processes.
3. It evaluates the implications of FCE results when there is a high variance in expert opinions.

We examine the reasons why the results of optimization and FCE diverge. The knowledge gained is useful for discovering weaknesses in the FCE results, as well as understanding areas of strategic miscalculations when FCE is employed.

Chapter II introduces the detailed process behind FCE by dissecting and explaining the method in its constituent components. Chapter III compares and contrasts the FCE method with an optimization model in choosing the most cost effective logistics platform for a combat logistics resupply scenario. Chapter IV reviews results and findings from Chapter III and presents a new method to test the effect of bias in expert opinions on FCE results. Last, Chapter V concludes with insights and suggestions for applying the formulation and methods developed in Chapter IV.

II. THE FUZZY COMPREHENSIVE ANALYSIS (FCE) METHOD

FCE identifies its roots in two mathematical approaches, Fuzzy logic and AHP. Sections A and B describe the main components while Section C combines them in a discussion of FCE step by step.

A. FUZZY LOGIC

In 1965, Lotfi A. Zadeh, a professor at UC Berkeley, revolutionized the idea of set theory and conventional logic by introducing fuzzy logic in his paper "Fuzzy Sets." Fuzzy logic is a form of probabilistic logic that deals with approximate inferences or ambiguous data (qualitative data) versus fixed or exact reasoning (binary yes/no choices and quantitative data) associated with conventional set theory. Professor Zadeh (personal communication, May 9, 2011) explained fuzzy logic as a, "logic of classes which do not have sharply defined boundaries." The concept behind fuzzy set theory involves a determination of whether an object, number, or quality is a "member" and the extent or "degree of truth" (degree of confidence) that it is within a set (Zadeh, 1965). The degree of truth is a value between 0 and 1, rather than only 0 or 1 as in classical logic.

The pejorative nature of the word "fuzzy" did not lend itself well in U.S. and English-speaking academic circles when it initially debuted in 1965. Additionally, its non-binary and imprecise way of thinking was criticized. There were some who did not believe in the partialities of truth presented by fuzzy logic or the use of "linguistic variables" (variables whose values are words) based on individual perception. Zadeh (personal communication, May 9, 2011) said that it is possible to compute with words, "you can add small to large… multiply them, etc." because it can be "useful to use words instead of just numbers." "Fuzzy Thinking" is a kind of scientific permissiveness based on imprecise results and subjective outcomes. However, some find this property especially useful when dealing with ambiguity in responses or situations involving imprecision. An example of ambiguity is when a statement is neither "true" nor "false".

Conventional set theory does not capture ambiguities of reality well. For example, conventional theory can only classify the lethality of a missile as either lethal or not. However, any competent decision-maker would want more information. More statistics on the degree of lethality would be valuable. Or if the missile is not lethal, how would this compare to another non-lethal missile? Clearly, both should have varying degrees of non-lethality. Problems such as this, where approximate classifications from subject matter experts provide better clarity to undefined phenomenon, is where fuzzy set theory lends itself. Fuzzy logic focuses on the subjectivity and imprecision found in human judgment. Stochastic methods and statistical analysis fall short in adequately handling the subjectivity and imprecisions of decision support processes. Fuzzy data can provide accurate insight by capturing the partialities associated with real human knowledge in decision-making (Yeh, Deng, & Pan, 1999).

Fuzzy math differs from conventional math in the same manner as fuzzy logic is to conventional logic. In the missile example, fuzzy math does not categorize a missile as lethal with True or False (1 or 0). Instead fuzzy math categorizes a missile's lethality with varying degrees of confidence or 'degrees of truth'. For instance, if a value of 0.00 is considered False and 1.00 is considered True, then a statement saying, "Missile A is lethal." can have a truth or confidence value of 0.8 and false value of 0.2. Moreover, the concept of a fuzzy set is important in understanding applications of fuzzy math. A conventional set is merely a collection of numbers or objects. On the contrary, a fuzzy set is a list of values, which describe to what degree an object belongs to a variety of characteristics or criteria.

B. ANALYTICAL HIERARCHY PROCESS

The weighting of criteria and sub-criteria greatly affects the accuracy of fuzzy evaluation results (Li, Jiang, Li, & Mu, 2011). Generally a methodical evaluation process that can properly handle the prioritization of evaluation elements for a decision objective is invaluable. AHP examines the relative importance of each criterion and determines their relative weighting. There does not seem to be a unified method for determining weights for criteria. In practice, various methods are used. Some interesting ways for weighting are entropy weights, AHP, and even direct weighting from the decision-maker (Yan, Zhang, Zhang, & Wu, 2008; Yeh et al., 1999). The concept of entropy weights involve weights established based on the probability a realization will occur. Thus, the higher the probability a realization will occur, the higher its entropy weight will be. However, this thesis focuses on AHP and direct weighting from the decision-maker.

AHP has been widely used in solving many complicated decision-making problems (Cheng, 1996; Cheng, Yang, & Hwang, 1999; Dağdeviren, Yavuz, & Kılınç, 2009). In the 1970s, Thomas L. Saaty developed AHP and explained its concepts in the following way.

> A basic premise of AHP is its reliance on the concept that much of what we consider to be “knowledge” actually pertains to our instinctive sense of the way things are. This would seem to agree with Descartes’ position that the mind itself is the first knowable principle. The AHP therefore takes as its premise the idea that it is our conception of reality that is crucial and not our conventional representations of reality by such means as statistics, etc. (Saaty, 1988, p. 110)

AHP is a type of systematic evaluation model called a multiple criteria decision-making (MCDM) method. There are different types of MCDM processes, but all handle problems of subjectivity, ambiguity, and uncertainty involved in a multi-criteria selection problem (Dağdeviren et al., 2009). AHP addresses the relative importance of each criterion over another involved in a decision problem with the purpose of determining criteria weights. AHP decomposes a problem, process, or course of action into the important factors that describe and form the overall evaluation. Criteria are decomposed into sub-criteria and the decomposition continues until a set of factors is established that can be directly evaluated. The evaluation set can consist of qualitative criterion values (i.e., lethality, reliability, attractiveness, stealth) requiring subject matter expert opinions to determine the relative importance of each to one another. Additionally, the evaluation set can contain quantitative (fuel economy, speed, cost, effective ranges, etc.) criterion values taken from test and evaluation sources or measured performance data. Therefore, AHP permits integration of intangible qualitative criteria alongside tangible quantitative criteria (Badri, 2001). Ultimately, AHP reconciles decision-maker’s or subject matter experts’ preferences (subjective inputs) for criteria and identifies the reliability of the preferences.

AHP consists of three steps: first, determine a set of evaluation elements or criteria and organize them into a hierarchy tree for analysis; second, assess pairwise comparisons of elements at each level of the hierarchy to formulate criteria and sub-criteria weights; and third, synthesize the prioritizations of the alternatives through combining normalized max eigenvalues and criterion weights.

Decision models are generally complex consisting of many decision elements or criteria with associated sub-criteria involved in the decision process. AHP's hierarchy tree aids the decision-maker by organizing the complexities of a decision. Specifically, once the decision-maker establishes a criteria set, AHP organizes these contributing elements into an interrelated hierarchical structure called a hierarchy tree. Following the previous missile example, Figure 1 details a simple hierarchy tree for a scenario with three alternative missiles.

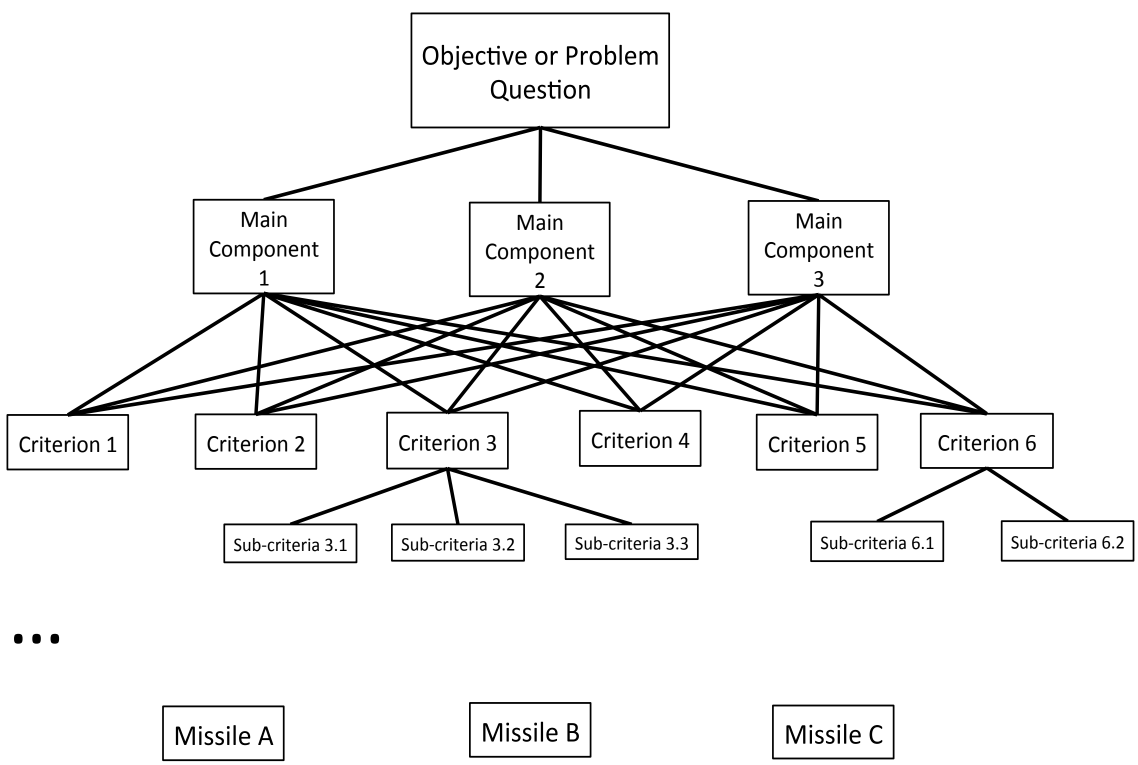

Figure 1. Sample AHP hierarchy tree for alternative missile system selection problem

The hierarchical decomposition of real events is a logical method for individuals to deal with the complexities of reality. The purpose of the hierarchy tree is to make criteria and sub-criteria evaluations easier across any given level to some or all criteria in upper levels. Additionally, the hierarchy tree ensures consideration is given to all aspects of a decision problem by mapping the problem's criteria in the tree. As seen in Figure 1, the hierarchy tree is similar to a family tree with parent nodes represented by the main components involved in a given decision objective. Child nodes branch off parent nodes organizing potentially multiple levels of criteria and sub-criteria. The criteria and sub-criteria help evaluate each decision alternative or course of action. A typical AHP hierarchy has a minimum of three levels: overall goal, objective, or decision of the problem at the top of the tree, multiple criteria with sub-criteria defining the alternatives in the middle, and decision alternatives at the bottom (Dağdeviren et al., 2009).

The initial estimates of criterion priorities (weights) are based on pairwise comparisons of criteria. Table 1, depicts a sample paired comparisons for the three sub-criteria of "Criteria 3."

	Sub-Criteria 3.1	Sub-Criteria 3.2	Sub-Criteria 3.3
Sub-Criteria 3.1	1	5	7
Sub-Criteria 3.2	1/5	1	3
Sub-Criteria 3.3	1/7	1/3	1

Table 1. Sample pair-wise comparisons of Criteria 3's sub-criterion

Subject matter expert opinions rank one criterion over another to create pairwise comparison matrices (similar to Table 1) at each level of the hierarchy tree. Table 1 displays one subject matter expert's opinion of how they feel each sub-criterion ranks in importance to the others. The values from the pairwise matrices come from Saaty's 1-9 scale in Table 2. Each value in the 1 through 9 scale has associated verbal descriptions. The pairwise matrix is read as follows. If the a criterion is evaluated with itself, they would be equally as important to each other and given the value one. Otherwise, referring to the criteria of the first entity (vertical criteria) and the second entity (horizontal criteria) for differing criteria would produce something different. The decision-maker determined the first entity, sub-criterion 3.1, is strongly more important than the second entity, sub-criterion 3.2 thus a value five was given. Understand that the values below the diagonal of 1's are simply the reciprocal of its counterpart. So in the case of sub-criterion 3.1 being strongly more important than sub-criterion 3.2, conversely sub-criterion 3.2 is strongly less important than sub-criterion 3.1. However, as with any subjective input, potentially inconsistent judgments from experts may occur.

Comparison Value	Description
1	The entities are of *equal* importance
3	The first entity is *weakly* more important than the second
5	The first entity is *strongly* more important than the second
7	The first entity is *very strongly* more important than the second
9	The first entity is *absolutely* more important than the second
2, 4, 6, 8	Intermediate or compromise values between the above categories
1/3, 1/5, 1/7, 1/9	Reciprocals are used to represent inverse relationships, e.g. 1/3 indicates that the *second* entity is weakly more important than the *first.*

Table 2. Verbal description of 1 to 9 scale (from Zuiliang. et al., 1993)

Fortunately the AHP provides an approach for improving consistency. A "consistency ratio" (CR) effectively reduces the introduction of bias from decision-makers (Badri, 2001). If the consistency ratio is less than or equal to 0.10, then the subjective inputs that formulated the weights for criteria are considered appropriately valid or "adequately consistent." Equally, if the CR is greater than 0.10, then this means that the pairwise comparisons must be repeated until a consistent matrix is obtained.

The maximum lambda, or maximum eigenvalue, of a paired comparison matrix is needed to solve for the consistency index (CI). The CI is calculated from Equation (1), where *n* is the number of criteria:

$$CI = \frac{(\lambda_{\max} - n)}{n} \quad . \tag{1}$$

The CI is used to calculate the CR. The CR determines if a paired comparison matrix is sufficiently consistent to evaluate criteria weights when CR is less than or equal to 0.1. Equation (2) details how to calculate the CR:

$$CR = \frac{CI}{R(n)} \quad (2)$$

where *R(n)*, is the random index. It is a known value dependent on the size of the matrix. For example, *R(n)* value for a 6x6 matrix is 1.24 (Hahn, n.d.).

In addition to ensuring the consistency of a decision matrix, AHP has two modes for synthesizing the prioritization of alternatives: the distributive and ideal modes. The distributive mode allows for violations in the deductive process of logic (Saaty, 1988). Consider a decision-maker who chooses Missile A over Missile B and Missile C. However, using the distributive mode the "inclusion of irrelevant alternatives," or decoy alternatives, can cause rank reversal when new alternatives are presented in the presence of old alternatives. This means that by presenting an irrelevant alternative, Missile D, the decision-maker may now choose Missile B. The distributive mode is also useful when there is dependence on the number of alternatives present. The ideal mode is the form of synthesizing priorities where rank reversal is not allowed. This is because the ideal mode preserves rank within its calculations. The ideal mode method divides the normalized values of the alternatives for each criterion by the value of the highest rated alternative before multiplying these priorities with local weights of each criterion. In this way, the addition of any new alternative that may dominate everywhere cannot cause rank reversal amongst existing alternatives (Saaty, 2001).

Note that FCE replaces the above third step with a fuzzy composition method (explained in detail later) to synthesize or produce fuzzy prioritizations of the alternatives. Thus, in FCE, the AHP is only used to organize elements of a decision into a hierarchy tree and formulate criteria and sub-criteria weights.

C. FCE

Fuzzy logic and AHP combine to construct the basis for the fuzzy comprehensive evaluation method used in this thesis. FCE is a five-step process. The first steps are based on AHP and the rest incorporates a fuzzy composition method that aggregates criteria weights and grade scores to produce an overall evaluation score for each alternative. The highest ranked alternative is provided to the decision-maker from the resulting fuzzy evaluation. The following is an overview describing the five steps. Chapter III provides a detailed exercise of the FCE method.

1. Determine the Criteria Set and Develop the Evaluation Hierarchy

A decision question of "Which missile system is best to employ against surface ships?" can be mapped with corresponding criteria and sub-criteria as seen in Figure 1, the sample hierarchy of alternative missiles.

2. AHP and Determining Element Weights

AHP estimates the additive utility weight for the criteria that best match the initial information provided by the decision-maker. AHP uses decision-maker or subject matter expert panel's opinions to perform paired judgments starting at sub-criteria and then moving up to corresponding criteria on concurrent levels to determine sub-criteria and criteria weights for the whole decision problem. The pairwise judgments form prioritizations of criteria and formulate criteria weights.

3. Establish Linguistic Grade Set and Evaluation Set

This step results in the development of a method for how each alternative should be evaluated for each criteria starting from the lowest level of the hierarchy. If expert opinions are incorporated due to qualitative criteria, then a linguistic grade scale is established. The result of this step forms the qualitative linguistic grade scale (excellent, good, fair, poor) and a way of mapping these qualitative scores onto a quantitative scale (excellent = 4, good = 3, fair = 2, poor = 1). The linguistic grades can be a five-point scale or any amount depending on what a decision-maker needs.

4. Evaluate the Alternatives' Fuzzy Judgment Matrix

Next determine the linguistic score for which an alternative satisfies a particular criterion. For example, the degree Missile A satisfies Criterion 1 may be judged as "excellent" while the degree Missile B satisfies Criterion 1 is "poor." This computation begins from the lowest level sub-criteria up to the highest criteria level creating a "subjective assessment matrix" for each alternative. An m x n subjective assessment matrix results where m is the number of criteria and n is the linguistic grade category (i.e., excellent, good, fair, and poor). Linguistic grades are useful in expressing the vagueness and partiality in expert opinions. Linguistic grade values facilitate the making of qualitative assessments.

Scaled probability matrices called fuzzy judgment matrices are calculated from the subjective assessment matrices. The scaled fuzzy judgment matrices are based on the total number of experts, *N*, who provided assessments. The fuzzy judgment matrix displays the scaled degree of membership within a linguistic grade that an alternative demonstrates in terms of each criterion. Refer to Chapter III, Section E for a detailed discussion.

5. Calculate Overall Synthetic Result

The overall optimal fuzzy evaluation is determined for each alternative. The overall fuzzy result is calculated by multiplying fuzzy judgment matrices for each alternative with criteria weight vectors. Furthermore, methods to select the "best" alternative when two are within the same category are applied to resolve ties.

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III. FCE IN MILITARY LOGISTICS

A notional logistics resupply example from the Joint Campaign Analysis (JCA) course at the Naval Postgraduate School provides the framework to study the implications of using FCE versus a mixed integer linear program (MILP). The MILP was originally implemented to solve a multi-criteria decision problem in the JCA course. We demonstrate the FCE method on the same scenario to study the FCE approach and compare its results to the MILP. This notional example is for research purposes only.

The following chapter first presents comparison of the U.S. favored approach in optimization to the foreign preferred FCE. We next discuss the background and modeling assumptions of the motivating scenario. We then describe of the MILP implementation from the original study. Followed by a thorough application of the FCE method performed on a combat logistics scenario. The chapter concludes with insights and remarks on how FCE performed and a comparison of results from both methods.

A. OPTIMIZATION VS FCE

In the following study, the MILP solves the logistic problem through an optimization approach. The optimal solution is determined through the minimization of an objective function centered on the operating costs of each alternative. Additionally, the objective function is constrained by the number of available logistic means of transportation (MOT), the number of trips each can make within a 50-hour cycle, and cargo limitations to meet the demand of warships.

The MILP implies a requirement for certain preciseness in the data. In order to accomplish this, a stochastic optimization is most appropriate. In this situation, one or more input parameters are subject to randomness in order to capture the uncertainty and imprecisions of real-life phenomena. The focus of this thesis is not on the effectiveness of preferred U.S. methodologies. Therefore, we did not pursue a stochastic optimization program. The intention of highlighting a stochastic optimization program is to demonstrate the weaknesses and strengths of optimization for comparison to the alternative, FCE. Knowledge of the parameters' distribution is also required to simulate the randomness of a parameter in optimization. This makes the optimization approach data intensive because larger sample sizes of data provide better estimates of the data distribution.

FCE solves ambiguous and imprecise information in the combat logistics problem through human knowledge for each logistic ship. Explicitly, FCE takes subject matter expert opinions as input data and through the FCE transforms them into quantitative output. FCE and optimization seem to be diametrically opposed methods. One method approaches problems subjectively (FCE) while the other approaches the same problem objectively (MILP). In terms of providing policy analysis for DoD purposes, FCE would prosecute the "best" policy alternatives with influences from policy-makers. However, optimization would determine the optimal solution subject to performance data and constraints of alternative policies. Selecting which methodology to employ depends on the area of interest and purpose of the user.

B. SCENARIO

The current concept of operations for the U.S. Navy's combat logistics supply process is to employ large, slow moving, and minimally defended logistical ships such as the USNS Lewis and Clark Class auxiliary cargo (K)/ammunition (E) ship (T-AKE). Additionally, the Navy faces an inability to resupply missiles at sea because there are no resupply platforms or procedures for this capability. A potential solution to these limitations is implementing the concept of a Mobile Landing Platform (MLP) as a resupply point at sea. An MLP is essentially a small highly mobile barge that is capable of replenishing a Surface Action Group (SAG) with supplies and strike missiles. The objective of this scenario is determining the most cost effective naval combat logistic force (CLF) mix to resupply a "Repositionable Expeditionary Basing" concept where the MLP operates as a repositionable expeditionary base (REB).

A "War at Sea" strategy is necessary to combat heightened hostilities from the Chinese with their claim to majority of the South China Sea (SCS). The "War at Sea" strategy implements a credible force structure with associated logistics, deployment strategy, and alliances to effectively deter Chinese aggression in the SCS. The repositionable basing concept merely encompasses an option for logistical solutions within the endeavors of the overall strategy. The overall goal is to determine which of three logistics MOT are the most cost effective to employ in this scenario. Specifically, how do we minimize the cost associated with resupplying a SAG and Flotilla fleet during conflict? The Repositionable Expeditionary Basing concept initially provided a "Blue" optimized analysis for a U.S. logistics force supporting the "War at Sea" strategy. We take this logistics analysis a step further by using Chinese analytical methods such as FCE to seek the "Red" solution in the following case study.

C. MODELING ASSUMPTIONS

REBs are flexible locations within a high-risk area (subject to attack) where logistic ships offload supplies and U.S. combat ships receive supplies. Figure 2 illustrates the proposed resupply process with the transit scheme between a transfer point and a single repositionable base. Any REB can be either close (500NM) or far (between 500NM and 1000NM) away from a transfer point node. But initially a T-AKE provides supplies to the model, at a transfer point. Transfer points are always reasonably in safe areas (port or prescribed friendly locations) clear from potential attacks. The transfer points are start nodes, where alternative logistic MOTs move supplies into the system. The alternative MOTs are the vehicles capable of transiting in and out of an area of threat or "risk" to get to a REB. All MOTs offload only two types of cargo: fuel and missiles. Each logistic MOT's transit speed is the same (15knots) towards a REB (blue arc) in order to reduce their detection and blend in with surrounding merchant ships and local fishing vessels. However, once a logistical MOT unloads its supplies, the assumption is that they are detected and must expeditiously exit the 'area of risk' (red arc) at max speed (sprint speed) back to the transfer point to restart the cycle. Once the REB receives demanded supplies, the SAG ships or Flotilla fleet (green arrows) can resupply from them as needed.

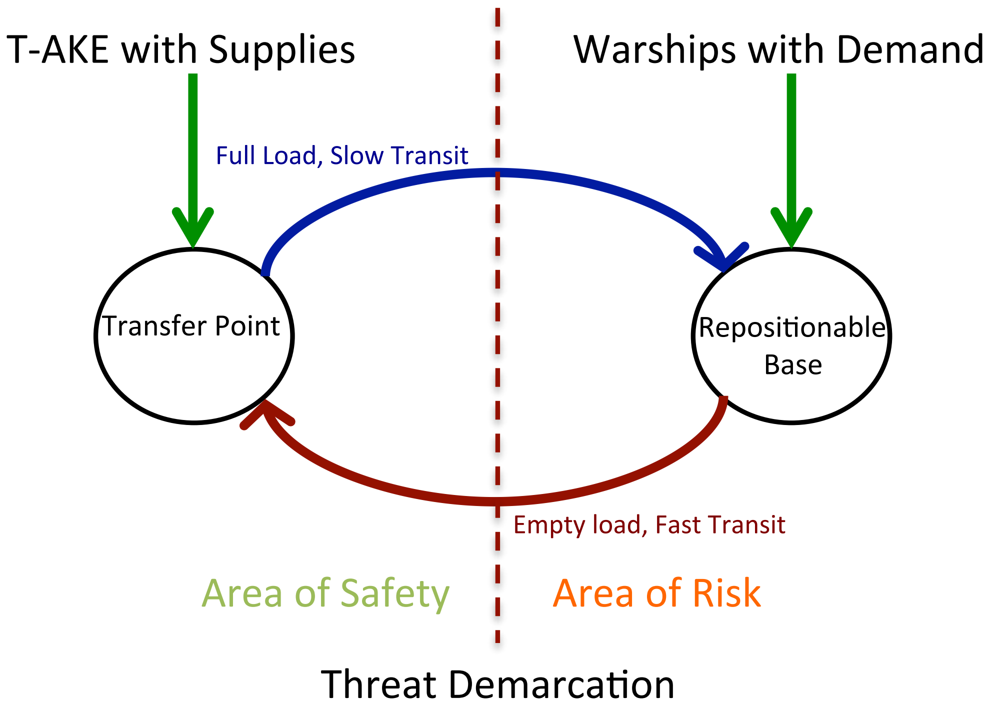

Figure 2. Transit scheme between a transfer point and a single base.

Figure 3 depicts the logistic network we analyze. A transfer point with alternative logistic ships on the left provide supplies to four possible REB locations on the right. Although Figure 3 illustrates one transfer point, multiple transfer points are possible subject to operational needs. The REB locations represent the demand nodes for the logistics network. Their locations are arbitrary. However, two REBs must be within 500NM of a transfer point (close) and two REBs between 500NM-1000NM (far). The location requirements for the REBs within an Area of Responsibility (AOR) are to provide redundant and flexible resupply points for combat operations. Conceivably more REBs can be placed "close" or "far."

The logistics network evaluates the performance of combinations of logistic MOT in determining the most cost effective logistics force mix. The thicker lines symbolize higher supply capacities because more trips can occur on them. The lighter lines depict lower supply capacities due to less number of trips made to further REBs.

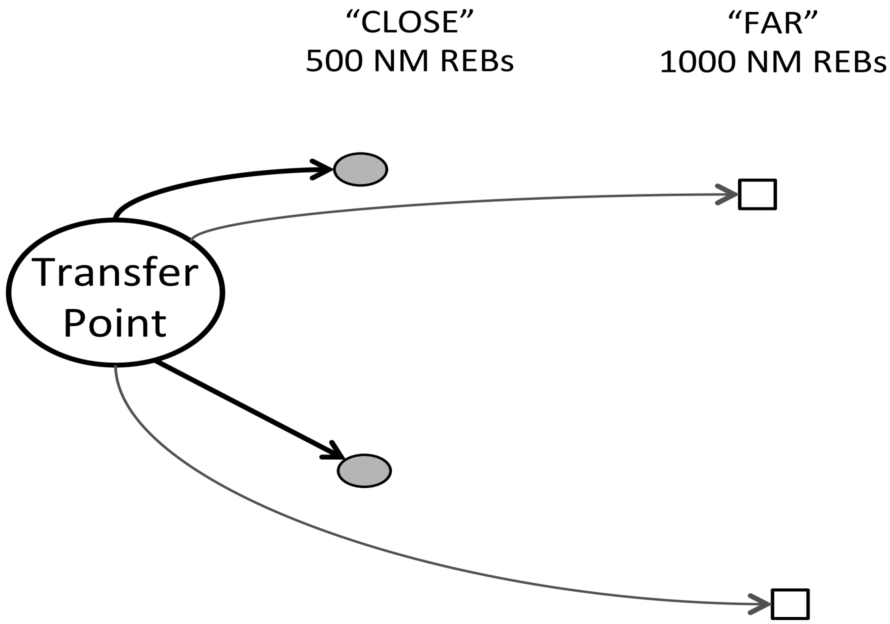

Figure 3. Disposition of possible REB locations to a transfer point.

The assets requiring supplies are SAG, consisting of three destroyers, and a Flotilla fleet of 34 small combatant ships. Notional fuel and missile requirements required for one days' conflict are used to estimate demand. Parameters for each logistic alternative were researched to determine capabilities. The three alternatives investigated for logistic resupply are the LCS-2 (Independence variant Littoral Combat Ship) and two modified versions of the Joint High-Speed Vessel (JHSV). JHSV refers to the version with lower fuel capability and higher missile capability. Combat Logistic Ship 5000tons (CLS-5000) is the modified JHSV with higher fuel capacity. Since the CLS-5000 is a concept in development, it provides potential design modifications to current JHSV capabilities and missions. Additionally, both the LCS-2 and JHSV are the latest ships employed by the U.S. Navy. Their missions and capabilities are still in question. Thus, this study was meant to explore potential logistics employment opportunities for both models. All unit parameters are defined for a 50-hour time step. Table 3 displays each alternative's parameters for this study.

Unit Parameters	CLS-5000	JHSV	LCS-2
Transit Speed (kt)	15	15	15
Sprint Speed (kt)	35	35	40
Assumed Fuel Tanker Capacity (ST)	335	126	151
Missile Capacity (ST)	400	600	231.5
P(Kill)	0.3	0.3	0.15
Operating Cost	$ 27,770,833.33	$ 21,500,000.00	$ 58,700,000.00
Reliability	0.75	0.75	0.8

Table 3. Alternative logistical MOT parameters.

According to Austal, the manufacturers' specifications for sprint speeds and missile capacities for the LCS-2 and JHSV are detailed in Table 3 (*Joint High-Speed Vessel & Littoral Combat Ship*, n.d.). Transit and sprint speeds are combined into "sprint speed" within FCE analysis because since the values for transit speed are the same across all MOTs, its inclusion is irrelevant. On the other hand, transit speed performs an important function in the optimization model. Transit and sprint speeds capture the time a logistic MOT travels slowly to a REB and rapidly back to the transfer point after delivering supplies within 50 hours. Missile capacities were based off maximum payload capability of each vessel. Fuel tanker capacities equal 20% of the current maximum fuel capability of each alternative. Operating costs for the LCS-2 and JHSV are 10% of total unit program costs taken from the U.S. Government Accountability Office's (GAO) *Assessments of Selected Weapons Programs* review (Sullivan, 2013). However, the CLS-5000's operating costs are calculated from the JHSV's operating cost aggregated due to upgrades of JHSV to CLS-5000 capabilities.

In maintaining simplicity and thesis goals, modeling lethality and reliability (how many hits can an alternative sustain) were not the focus of the optimization model. However, the opportunity to explore these effects of increased defensive capabilities or reliability of an alternative to complete its mission (maintenance schedules, operating readiness of crew, etc.) is provided within the "probability of kill" (P(Kill)) and reliability criteria and left for further study. Both parameters were not incorporated into the MILP because its inclusion would have taken the MILP beyond the scope of this case study and made comparisons to FCE difficult. However, FCE's incorporation of them was appropriate in capturing the factors affecting each alternative's rankings. The P(Kill) values found in Table 3 are provided based on the unclassified defensive capabilities of each ship. The reliability values are estimated from author experience given that reliability is fairly variable in the real world.

D. OPTIMIZATION MODEL

In answering the research question of determining the most cost effective logistics force mix to employ in this scenario, we formulate a MILP, implement it in MS Excel, and solve it using the default MS Excel MILP Solver.

The following definitions and equations explain the optimization model as we attempt to satisfy combat operational demands while minimizing operating cost.

1. Sets:

$i \in I$ ={CLS-5000, JHSV, LCS-2}	vessel type
$j \in J$ ={fuel, missiles}	cargo type
$k \in K$ ={500NM, 1000NM}	distance REB is located
$r \in R$ ={Close, Close to Close, Far}	route type

2. Data [units]:

c_i = operating cost of vessel type *i* [*$*]

$a_{i,j}$ = capacity of each vessel type *i* for cargo *j* [*ST/trip*]

$t_{i,k,r}$ = trips vessel *i* makes to distance *k* on route *r* [*trips/vessel*]

m_k = number of bases at distance *k* [*bases*]

$d_{j,k}$ = demand for cargo *j* at distance *k* [*ST/base*]

s_i = number of available vessels of type *i* [*vessel*]

3. Decision variables:

$X_{i,r}$ Integer (≥ 0) Number of vessel type *i* used on route *r*

4. Formulation:

$$\min_{X_{i,r}} \sum_{i \in I} \sum_{r \in R} c_i X_{i,r} \tag{3}$$

s.t. $$\sum_{i \in I} \sum_{r \in R} a_{i,j} t_{i,k,r} X_{i,r} \geq m_k d_{j,k} \qquad \forall\, j,k \tag{4}$$

$$\sum_{r \in R} X_{i,r} \leq s_i \qquad \forall\, i \tag{5}$$

$$X_{i,r} \geq 0 \textit{ and integer} \qquad \forall\, i,r \tag{6}$$

5. Discussion

Equation (3) is the objective function. It measures the cost of employing vessels. Table 4 details the specified values for the parameters of each alternative MOT for the MILP. The cycle length of 50 hours is based off the time to complete far trips. The time to complete round trips to close or far bases are determined from each alternative's transit speed to a REB and sprint speeds back to a transfer point. Only two close and two far REBs are modeled totaling four within an area of operation (AO). The total wartime missile and fuel demands, measured in short tons (ST) per hour, are required for each close and far base. This supports the redundancy of the system. If three out of the four REBs are eliminated, the remaining REB is capable of supplying all combat ships within the AO. Missile demands are hypothetically calculated for the SAG and Flotilla fleet. The fuel demands are calculated with the assumption that the SAG and Flotilla group always maintain 85% fuel capability. Therefore, they require about 1000ST of fuel to maintain operability according to operating procedures.

Constraints	Parameter	CLS-5000	JHSV	LCS-2
Cycle length		50 hours	50 hours	50 hours
Total_Number_Close REBs to resupply	m_k	2	2	2
Total_Number_Far REBs to resupply	m_k	2	2	2
Total Close base_Wartime_Missile_Demand [ST]	D_jk	1110	1110	1110
Total Far base_Wartime_Fuel_Demand [ST]	D_jk	2000	2000	2000
Number_Close_Trips	T_ikr	2.1	2.1	2.2
Number_Far_Trips	T_ikr	1.1	1.1	1.1
Available number	s_i	10	15	16

Table 4. Parameter values for optimization model

The possible routes a vessel can travel to provide supplies are as follows: two trips to close REBs or one trip to a far REB within 50 hours. The last route of only delivering to one close REB is to meet any leftover demand. Therefore, $T_{i,k,r}$ assumes a value of about one or two depending on whether a close or far trip is taken. The trip index, *k*, accounts for the worst-case scenario when the MILP selects an alternative to go close or far. For example, if a REB is 600NM away from a transfer point, the MILP assumes the worst case and calculates the length of this trip with a 1000NM distance. This means that $T_{i,k,r}$ can only make a maximum of one trip to this REB and assumes the value of one. Lastly, the available number of alternative ships is based on projected productions of each alternative by 2025. This was a stipulation from the JCA course where this example originated.

The first constraint set, Equation (4), ensures demand is satisfied. They allow for oversupply of the REBs without penalty. The second constraint set, Equation (5) limits the number of available vessels. Lastly, the decision variable is non-negative and integer as seen in Equation (6).

6. MILP Solution

The MILP found the most cost effective logistical force to resupply U.S. combat forces engaged in an Area of Operation (AO) in the S. China Sea was nine Combat Logistic Ships 5000tons (CLS-5000). Three CLS-5000 vessels were required to fulfill the demand of “close” REBs and six CLS-5000 fulfilled the demands for “far” REBs. There was no better combination of a CLF mix of logistics ships. The CLS-5000 provided the optimal solution in terms of cost and cargo capacity to resupply. Yet to verify the fidelity of these results and provide a ranking of the alternatives, the optimization model was run without the optimal choice of CLS-5000 as an option. The results were eight LCS-2 (seven required to supply “far” bases and one for “close” bases) and 15 JHSV’s (eight to supply “far” bases and seven to supply “close” bases) to fulfill what nine CLS-5000s accomplished alone. These results indicate that the ranking of the alternative methods of logistical transportation are 1) CLS-5000 2) JHSV 3) LCS-2 based on the number of ships employed to meet demand. The lower number of ships MILP selected equates to higher cost.

E. FCE MODEL

In this section we examine an FCE approach to the combat logistic resupply scenario. FCE is useful when human knowledge is necessary and human evaluations are needed due to the presence of incomplete or vague information. Each alternative MOT possesses differing capabilities or factors that render it useful for employment in a logistics mission. An advantage is that we already know most of the factors involved in this scenario from the MILP model. However, it is difficult to systematically evaluate how each alternative's capabilities meet a decision-maker's objective without expert knowledge and a method for analyzing performance data. FCE is a method that can suitably prioritize the alternatives to best meet a mission's intent. The following are the steps for this qualitative assessment.

1. Determine Criteria Set and Develop Evaluation Hierarchy

We use the following six criteria to evaluate the alternatives: fuel capacity, missile capacity, sprint speed, probability of kill, replacement cost, and reliability, are used to evaluate the alternatives. These elements make up the set, *E*, that are important in describing the qualities needed in a logistical MOT. In Equation (7), e_i represents the i^{th} evaluation element in the set E,

$$E = \{e_1,\ e_2,\ ...,\ e_n\}\,. \tag{7}$$

A hierarchy tree is formed organizing the criteria for each alternative. Figure 4 illustrates this study's hierarchy as it relates the criteria to the alternatives evaluated. At the top of the hierarchy is the objective and focus of this study. The second level consists of the six criteria. The last level displays the three alternatives and their relationship with each criterion.

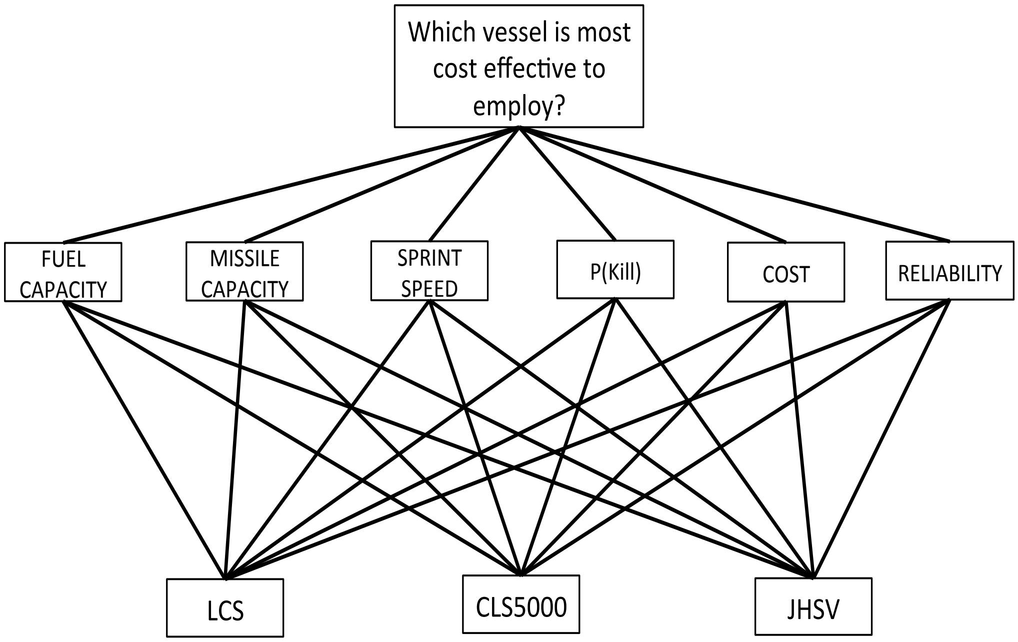

Figure 4. Logistic resupply hierarchy

2. AHP and Determine Element Weights

The second step is a comparative judgment of the alternatives based on each criterion by subject matter experts. This begins the determination of importance of each criterion among the rest within a level of the hierarchy. AHP uses Saaty's standardized scale from 1 to 9 for the multiple pairwise comparisons.

Through pairwise comparisons of each criterion (with exception of transit speed) against the rest, appropriate weights for each criterion are determined. Table 5 depicts potential values for the paired comparisons based on our expert judgment of the middle level of criteria illustrated in the logistic resupply scenario's hierarchy tree. These values would be determined by the decision-maker choosing the level of importance (from Saaty's 1-9 scale) for a criterion to another. The same criterion would be equally as important to itself and given the value one when compared to itself on the pairwise comparison matrix. Otherwise, referring to the criteria of the first entity (vertical criteria) and the second entity (horizontal criteria) for missile capacity would produce the following. In this case, the decision-maker determined the first entity, fuel capacity, is less than weakly more important than the second entity of missile capacity (highlighted gray in Table 5). On the other hand, values below the diagonal of 1's are simply the reciprocal of its counterpart value. So if fuel capacity is less than weakly more important than missile capacity, then missile capacity is more than weakly more important than fuel capacity (highlighted yellow in Table 5). Because this analysis focuses on the cost of operation, cost is generally more than strongly more important that any other criterion. The values in Table 5 were logically generated from our knowledge, research and experience of each alternative while we acted as the decision-maker. The purpose for generating the pairwise matrix, by acting as the decision-maker, is to incorporate real human knowledge and evaluations in preparation for the simulation robustness check discussed in Chapter IV.

Pairwise Comparison Matrix	Fuel Capacity	Missile Capacity	Sprint Speed	P(Kill)	Cost	Reliability
Fuel Capacity	1	2	4	3	0.167	4
Missile Capacity	0.5	1	3	2	0.143	3
Sprint Speed	0.25	0.33	1	0.5	0.11	1
P(Kill)	0.33	0.50	2	1	0.125	2
Cost	6	7	9	8	1	9
Reliability	0.25	0.33	1	0.5	0.11	1

Table 5. Pairwise comparisons of criteria with fractions converted to decimals

After the pairwise comparisons are completed, the eigenvectors of the matrix are calculated. The largest real eigenvalues of each row's eigenvector form the max eigenvector of the matrix. Then, the normalized values of the maximum eigenvector formulate the weights in AHP. However, a second and equivalent procedure commonly used is calculating the geometric mean of each row of a pairwise decision matrix. Equation (8) describes how to calculate the weights through geometric means for the set of G. P is the n x n pairwise comparison matrix (Table 5) formed from subjective inputs:

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix}$$

$$G = \{g_1, g_2, ..., g_m)$$

$$g_1 = \sqrt[n]{p_{11} \times p_{12} \times \ldots \times p_{1n}} \tag{8}$$

and each row of P calculates a value for g_i.

Once all the g_i values are calculated for all i in $1 \ldots m$, they are normalized forming the criteria weights. Equation (9) demonstrates how the criterion weight for fuel capacity is calculated:

$$g_1 = \sqrt[6]{1 \times 2 \times 4 \times 3 \times 0.167 \times 4}$$

$$g_1 = 1.587$$

$$w_1 = \frac{1.587}{\sum_{i=1}^{6} g_i} \tag{9}$$

for all g_i.

Table 6, displays the resulting weight vector for each criterion to the overall objective given expert inputs.

4. Weight Vector	Fuel Capacity	Missile Capacity	Sprint Speed	P(Kill)	Cost	Reliability
Weight	0.17	0.11	0.04	0.07	0.57	0.04

Table 6. Weight vector

The set of weights make up the weight vector, W, found in Equation (10):

$$W = \{w_1, w_2, \ldots, w_m\} \tag{10}$$

3. Establish Linguistic Grade Set and Evaluation Set

Four linguistic assessment terms were chosen for this study's evaluation set: excellent (v_1), good (v_2), fair (v_3), and poor (v_4). Thus, Equation (11), V, is the linguistic assessment grade vector used to score each alternative. The next section clarifies the role of V:

$$V = \{v_1,\ v_2,\ v_3,\ v_4\}\ . \tag{11}$$

4. Evaluate the Alternatives' Fuzzy Judgment Matrix

Evaluating each alternative's membership in a linguistic grade for a certain criterion forms the subjective assessment matrices. These evaluations begin from the lowest level child nodes of the evaluation hierarchy and results are aggregated up towards parent nodes as described in Chapter II. For this study, we provide the subjective judgments that grade the alternatives based on each criterion by pretending to be multiple (20) experts. Of the 20 votes, 10 votes were naïve opinions (inexperienced judgments) and the other 10 votes were subject matter expert judgments. The sources for expert opinions are gathered from studying personal experiences from LCS-2 operators and through research of the alternatives. Table 7 is the subjective assessment matrix as a result of the 20 evaluations.

Subjective Assessment Results						
Criteria/Grade	Excellent	Good	Fair	Poor	Row Sum	Logistic Ship
Fuel Capacity	5	8	6	1	20	LCS-2
Missile Capacity	0	4	9	7	20	
Sprint Speed	17	3	0	0	20	
P(kill)	16	4	0	0	20	
Cost	0	4	7	9	20	
Reliability	12	7	1	0	20	
Criteria/Grade	Excellent	Good	Fair	Poor		Logistic Ship
Fuel Capacity	13	6	1	0	20	CLS-5000
Missile Capacity	6	10	3	1	20	
Sprint Speed	1	3	7	9	20	
P(kill)	4	12	3	1	20	
Cost	10	7	3	0	20	
Reliability	11	6	3	0	20	
Criteria/Grade	Excellent	Good	Fair	Poor		Logistic Ship
Fuel Capacity	2	10	7	1	20	JHSV
Missile Capacity	10	7	3	0	20	
Sprint Speed	0	2	6	12	20	
P(kill)	5	12	3	0	20	
Cost	6	14	0	0	20	
Reliability	11	9	0	0	20	

Table 7. (Notional) Subjective assessment matrices

In Equation (12), for every e_j, f_{ij} represents the degree of membership on e_j to v_i, (i =*excellent, good, fair, poor*), based on subjective votes:

$$f_{ij} = \frac{n_{ij}}{N}. \tag{12}$$

In Equation (12), n_{ij} is the number of $e_j \in v_i$ (n_{ij} are the values found in Table 7); N is the total number of subjective votes (N =20).

Equation (13) is the scaled subjective assessment matrix of an alternative called a fuzzy judgment matrix, F, of element e_j, on grade value v_i, created from every f_{ij}:

$$F = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix}. \tag{13}$$

Table 8 is the fuzzy judgment matrices for the LCS-2, JHSV and CLS-5000. Essentially, it is a rescaled from the subjective assessment matrix based on the number of subjective votes for each e_j, on grade value v_i out of 20 votes.

Degree of Membership for Each Vessel						
Criteria/Grade	Excellent	Good	Fair	Poor	Row Sum	Logistic Ship
Fuel Capacity	0.25	0.4	0.3	0.05	1	LCS-2
Missile Capacity	0	0.2	0.45	0.35	1	
Sprint Speed	0.85	0.15	0	0	1	
P(kill)	0.8	0.2	0	0	1	
Cost	0	0.2	0.35	0.45	1	
Reliability	0.6	0.35	0.05	0	1	
Criteria/Grade	Excellent	Good	Fair	Poor	Row Sum	Logistic Ship
Fuel Capacity	0.65	0.3	0.05	0	1	CLS-5000
Missile Capacity	0.3	0.5	0.15	0.05	1	
Sprint Speed	0.05	0.15	0.35	0.45	1	
P(kill)	0.2	0.6	0.15	0.05	1	
Cost	0.5	0.35	0.15	0	1	
Reliability	0.55	0.3	0.15	0	1	
Criteria/Grade	Excellent	Good	Fair	Poor	Row Sum	Logistic Ship
Fuel Capacity	0.1	0.5	0.35	0.05	1	JHSV
Missile Capacity	0.5	0.35	0.15	0	1	
Sprint Speed	0	0.1	0.3	0.6	1	
P(kill)	0.25	0.6	0.15	0	1	
Cost	0.3	0.7	0	0	1	
Reliability	0.55	0.45	0	0	1	

Table 8. Fuzzy judgment matrices

5. Calculate Overall Synthetic Result

Equation (14) is the fuzzy composition method used to calculate the overall synthetic results. The vector of criteria weights, W, are multiplied to the fuzzy judgment matrix of each alternative:

$$Y = W \times F = \{w_1, w_2, \ldots, w_m\} \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix} = \{y_1, y_2, \ldots, y_n\} \ . \quad (14)$$

The Y is a fuzzy vector representing all evaluation elements' contributions and the degree of membership of each alternative for a grade value. Table 9 displays the distribution of the overall synthetic results for each alternative. Additionally, the modes of the overall fuzzy results of each alternative are highlighted in yellow. Note that for the LCS-2 there is almost maximum equal membership within the fair and poor grade values. This presents areas of miscalculation especially when an alternative has higher membership in a higher-grade category and just a little less but similar membership in the lower categories. Only the mode is taken as the overall result. No consideration is given to the distribution of grade scores. The "Insights and Conclusions" section of this chapter provides more discussion on this issue.

FUZZY EVALUATION OF LOGISTIC SHIPS	Excellent	Good	Fair	Poor
FUZZY VECTOR (LCS-2)	0.158	0.237	0.301	0.304
FUZZY VECTOR (CLS-5000)	0.465	0.364	0.142	0.028
FUZZY VECTOR (JHSV)	0.283	0.585	0.097	0.034

Table 9. Overall synthetic evaluation results

The CLS-5000, modified JHSV won as the best candidate to supply warships engaged in the S. China Sea. The CLS-5000 had the highest quality score in the excellent grade. The remaining rankings are as follows: 2) JHSV 3) LCS-2. The MILP model also resulted in the same rankings.

F. INSIGHTS AND CONCLUSIONS

Both FCE and the optimization program produced the same results of selecting the CLS-5000 as the most cost efficient logistics vehicle. FCE handled the imprecision and subjectivity of reality well. FCE captured the same results as a robust mathematical MILP, but through the use of subject matter expert judgments. Yet, it is not free of weaknesses. FCE models the imprecision and subjectivity but does not provide techniques to contend with bias. Due to the subjectivity of the FCE method, the results are only as good as the expert inputs. Moreover, decision-makers can artificially weight inputs from senior experts over junior experts simply due to seniority, not due to who is more knowledgeable. Thus, for results to be more accurate, it is necessary to obtain opinions from a sizeable pool of experts, *N* (number of experts), in order to deal with the inherent bias of reality. Chapter IV focuses more on this issue in FCE.

Another pitfall of FCE is how the overall results are determined. FCE only considers the mode of results as the final score for an alternative. FCE does not consider the distribution of each alternative's resulting membership value across the linguistic grades. If an alternative has 51% membership in the excellent category and 49% in the poor, according to AHP the alternative is still considered excellent. The methods for breaking ties are problematic as well. During this study, situations arose when both methods for breaking ties (Max Degree of Membership or Ordered Weighted Averaging) provided contradicting results. Table 10 demonstrates the difficulty in separating the best alternative amongst two alternatives evaluated within the same category. Unfortunately, no clear guidance is available to handle this.

FUZZY EVALUATION	Excellent	Good	Fair	Poor
FUZZY VECTOR (LCS-2)	0.281	0.251	0.249	0.219
FUZZY VECTOR (CLS-5000)	0.4235	0.3685	0.157	0.051
FUZZY VECTOR (JHSV)	0.2905	0.5145	0.1435	0.0515

Table 10. Fuzzy evaluation with alternatives within the same category

LCS-2 and CLS-5000 tie for having the highest-grade membership in "excellent." Two methods are commonly used to break ties, "Max Degree of Membership" and "Ordered Weighted Average." The Max Degree of Membership method takes the alternative with the highest degree of membership in a grade as the winner. Thus, the CLS-5000 is the winner according to Table 10. The Ordered Weighted Averaging method determines the better alternative by aggregating an alternative's fuzzy vector with a value scale from 1 to 4, where excellent = 4, good = 3, fair = 2, poor = 1, as seen in Equation (15):

$$l = \{4,\ 3,\ 2,\ 1\}\ . \tag{15}$$

Equation (16) produces the final ordered weighted average for each alternative, X:

$$X = \frac{Y \times l}{4} = \frac{\{y_1,\ y_2,\ y_3,\ y_4\} \times \{4,\ 3,\ 2,\ 1\}^T}{4}\ . \tag{16}$$

CLS-5000 wins again with an ordered weighted score of 0.79 versus 0.65 for LCS-2.

Lastly, the ability of the decision-maker to provide direct weighting for criteria can be advantageous or disastrous. The criteria weighting can greatly affect the accuracy of the results. The resulting decisive alternative or course of action is sensitive to weighting changes of the criteria and sub-criteria. A slight change in the weight of a criterion can result in different decisions. Therefore, the accuracy of the criteria weights is paramount.

Chapter IV extends the logistics case study to investigate the effects of high variance in expert opinion. Specifically, the issue of expert bias is addressed by testing the robustness of expert opinions when a “rogue” expert is introduced.

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IV. HIGH VARIANCE IN EXPERT INPUT STUDY

In this chapter, we seek to determine how one “rogue” or adversarial expert can bias FCE results. Remember, FCE results are as useful as the subjective inputs given by experts. If an adversarial expert were capable of breaching enemy decision support processes, such as FCE, their goal would be to increase the miscalculations FCE provides. By inserting incorrect or inverse votes into the FCE system, we study how an adversarial expert can effectively bias and influence the results. We find that inexperienced or misinformed experts can have the same effect on the results as an adversarial expert.

The goal of this study is to test and verify the robustness in FCE results in the presence of misleading information using the SCS logistic resupply scenario from Chapter III. We formulate a program using the R statistical language to evaluate the uncertainty of FCE results and test the effect of expert bias FCE rankings. The “R2B” (robustness to bias) tester simulates *N* (number of experts) subjective matrices from the original subjective assessment matrices of 20 notional experts (from Table 7) in Chapter III. Out of the *N* experts, a person’s subjective assessment matrix is simulated from an adversarial standpoint. The adversarial subjective judgment matrices, for each alternative, are generated from reversing a given set of “true” distributions of opinions. Specifically the “true” distributions’ probabilities are switched as follows: excellent with poor and fair with good. These “true” opinions come from the original subjective assessment matrices that we know produce the CLS-5000 as the best alternative used in Chapter III’s case study. Effectively R2B simulates expert opinions from a distribution of opinions and compares the results to the “real” results. After a few parameters are initialized and set, running the R2B tester provides users the number of experts required to maintain accurate results for the particular scenario they are studying. The following chapter discusses the R2B tester algorithm in detail and reviews some key findings from this study.

A. "R2B" FUNCTION

1. Set/Initialize Parameters

R2B is an R statistical function that requires four arguments (*n, m, Nmax,* and *wts*). Figure 5 displays the R2B function's pseudo code.

```
R2B=function(n,m,Nmax,wts){
 Save_alt=matrix(0,nrow=Nmax,ncol=m,byrow=TRUE)
   for (N in 2 to Nmax){
     for (q in 1:100){
       SME_matrix<alt> = simSME(true_matrix,N)
       Normalize SME_matrix<alt>
       Fuzzy_matrix<alt> = FuzzyResults(wts,norm_SME<alt>)
       Normalize Fuzzy_matrices
       Save results as probabilities of the number of times an
         alt. lands in a linguistic category for each alternative
    }
  }
  print(results)
}
```

Figure 5. R2B pseudo code

The user must input the number of criteria (fuel capacity, sprint speed, cost, etc.), *n*, that the alternatives are evaluated upon. Input the number of linguistic categories, *m*, after establishing the linguistic grade values (excellent, good, etc.). Normally the decision-maker determines this. Currently *m* is set to four because the logistic scenario had four linguistic categories: excellent, good, fair, and poor. *Nmax* is a variable describing the maximum number of experts to evaluate. R2B has the default setting of *Nmax* equal to 10. This means that a maximum of 10 experts are evaluated. A user is able to find the effects of one rogue expert amongst any number of experts between two up to a maximum 10. This default setting reflects the difficulty in amassing large numbers of experts at once. However, dependent on the scenario presented to R2B, it is possible that more than 10 experts are required for conclusive results in the presence of one rogue expert. The last argument, *wts*, is the AHP criterion weights. Once the weights are determined by either AHP, either given directly from the decision-maker, or taken from a sample problem, set the *wts* parameter to those values.

2. Import "True" Fuzzy Subjective Assessment Matrices

The user must first import the original, distribution of fuzzy subjective assessment opinions as matrices into the R statistics package. Although not direct inputs into R2B, these true matrices are coded into the R2B algorithm to generate opinions. In order to do so, the true matrices are run through the *simSME()* function from the *GenerateSMEcrit.R* script within R2B. The *simSME()* function takes in the true matrix as its first argument and *N*, the number of experts to test, as the second argument. This function generates reliable, *N-1*, expert opinion matrices and one adversarial expert subjective matrix by switching the "true" matrix probabilities for excellent with poor and fair with good for each alternative.

3. Function Implementation

We initialize matrices with *N* rows and *m* columns to save the number of times a fuzzy evaluation result lands in any of the four linguistic categories. The results appear as normalized probabilities. Table 11 illustrates the expert statistics matrix for CLS-5000 in this study. 100 simulations were conducted for each row of Table 11. The results depict the probability of a fuzzy evaluation falling in a certain category starting with two expert's opinions up to 10. Table 11 says that when two experts are present with one that is misinformed or adversarial, CLS-5000 has a linguistic grade predominantly in the good category. Explicitly, CLS-5000 has 27% membership in the good category. But with more informed experts, we find unsurprisingly our original finding with the CLS-5000 predominantly in the excellent grade.

Experts	CLS-5000			
	EXCELLENT	GOOD	FAIR	POOR
2	0.22	0.27	0.23	0.28
3	0.43	0.36	0.18	0.03
4	0.51	0.3	0.19	0
5	0.63	0.31	0.06	0
6	0.58	0.39	0.03	0
7	0.67	0.29	0.04	0
8	0.65	0.33	0.02	0
9	0.6	0.38	0.02	0
10	0.72	0.28	0	0

Table 11. CLS-5000 saved fuzzy evaluation results by number of experts

The same steps of multiplying normalized subjective assessment matrices with criteria weights in FCE occur using the *FuzzyResults()* function depicted in Figure 5. The results for the 100 iterations of each *N* ranging from 2 to 10 are saved into a table such as Table 11 for each alternative for examination.

4. Results

Overall, CLS-5000 is again ranked the "best" alternative for a logistic MOT in Table 12. Table 12, depicts the results from the robustness test of FCE results to expert bias. This is not a surprising result since our "true" inputs put CLS-5000 as the best. The purpose is to check the variability in the results due to the presence of an adversary or misinformed individual. Variations mainly occur with lower numbers of experts. The results illustrate that an adversary in the presence of only one other opposing expert can skew the results in their favor. The combat logistics scenario requires at least two experienced experts out of three to assume useful results. Similarly, it is recommended that at least two times as many unbiased experts are required for one biased.

Additionally, the results reveal FCE can be a reasonably robust method even in the presence of intentional bias. The ranking for JHSV and LCS-2 are similar to those found in Chapter III. Second place is secured by JHSV predominantly in the "good" category. Lastly, LCS-2 teeters between "fair" and "poor." These values are scaled from each alternative's saved fuzzy evaluation results (as in Table 11) multiplied by row to a value scale for each linguistic category. The value scale is from 1 to 4 with excellent = 4, good = 3, fair = 2, and poor = 1.

Experts	WT AVG Scores		
	CLS	JHSV	LCS-2
2	0.57525	0.62375	0.6115
3	0.8265	0.7325	0.5875
4	0.844	0.81	0.47275
5	0.85	0.78	0.47
6	0.905	0.785	0.46
7	0.91075	0.77	0.4555
8	0.9175	0.77	0.445
9	0.9175	0.7675	0.47
10	0.9225	0.7625	0.4625

Table 12. Scaled simulated fuzzy evaluation scores from 2 to 10 experts

This study also found that in general with a greater number of experts providing inputs, the closer the results were to the true response. R2B is a way of evaluating the potential uncertainty in FCE and showing under what conditions a same alternative is still chosen. R2B results are specific and applicable only to the existing scenario. However, this method can be applied to any set of expert opinions to estimate the potential effect of bias on FCE results.

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V. CONCLUSIONS

A. SUMMARY

The FCE method is one of many that solve multiple criteria decision-making (MCDM) problems by incorporating quantifiable subject matter expert input. In the literature, there are many MCDM techniques; and some are better suited than others depending on the decision problem. Additionally, in practice, there is generally no standard for how to transform subjective inputs into numbers to prioritize a group of alternatives. Practitioners generally meld varying MCDMs with AHP or another engineered mathematical method to create a reasonable process for transforming expert opinions into quantitative data (Cheng et al., 1999; Dağdeviren et al., 2009; Yan et al., 2008). This thesis performs its transformations through AHP criteria weights and a fuzzy composition method. Cheng (1996) also uses a similar method to evaluate alternative missile systems using fuzzy AHP.

FCE is an appropriate decision aid capable of explicitly examining various complex events, decisions, or phenomena. Specifically, this thesis focuses on military applications. Potential and current adversaries, along with U.S. allies, all use FCE. FCE's prevalence in the U.S. seems to be growing too (Singh et al., 2013). The analysis conducted in this thesis has developed and highlighted potential pitfalls and benefits to FCE. The understanding of this knowledge can prove exceptionally useful for friendly or adversarial decision support processes.

For most of the last 40 years, many academics were quick to discredit the principles and methodology behind FCE. This is because FCE incorporated imprecise and subjective processes. Yet, as researchers continue to find uses for capturing the partialities and uncertainties in everyday phenomenon, FCE has become a notable methodology. But the subjectivity in inputs and the bias that could be present remains a criticism.

The weaknesses of FCE include the requirement for a well-structured decision problem. FCE does not perform well unless correct parameters of a problem are known.

Specifically, in order for the AHP in FCE to perform well, it requires a well-mapped hierarchy tree with "consistent" subjective inputs to formulate appropriate criteria weights. The subjective judgments and rankings from the decision-maker have a large influence on the accuracy of FCE results. Accuracy, in statistical terms, is the degree of closeness a measurement of a quantity is to its actual "true" value. If subjective judgments have large biases, then FCE results are unrealistic and expected to be inaccurate. Although AHP relies on a "consistency ratio" to reduce the amount of biased error in paired comparisons, the paired comparisons are not resistant to the predisposition of results by the decision-maker. Issues arise during group think tanks where expert opinions from higher ranked, less knowledgeable officials are accepted over more knowledgeable junior officers. Normally, a solution to this problem is the requirement for pairwise comparison matrices to be "consistent" with less than 10% of inputs being inconsistent. However, subjective judgments can still be skewed when younger experts are coerced into reevaluating their original subjective inputs in order to create consistent matrices towards decision-makers' predispositions. While these situations may occur rarely, this thesis explores what could happen under different scenarios where expert judgments are faulty.

FCE rankings are not precise without a method for choosing a larger sample size of subjective inputs involved in a particular scenario. FCE takes the mode of fuzzy evaluations as the solution and disregards the distribution of these evaluations. If the alternative has membership scores almost evenly distributed between all the categories from "excellent," "good," "fair," and "poor," only the category in which the alternative has the highest membership prevails. Additionally, FCE results are prone to potentially intentional bias. The R2B (robustness to bias) algorithm was created in the R statistical language to test the robustness of FCE results to expert bias through simulation. The range of experts was chosen based upon the realities of a limited availability of subject matter experts in general. R2B provides an estimate of the variability associated with FCE outcomes.

R2B's algorithm allows users to estimate the number of experts required in order to reduce the probability of an incorrect ranking for a particular scenario with one malicious expert. R2B cannot guarantee the accuracy of results. Accuracy in results is generally unknown, even with a rigorous study of where the original FCE results came from. Nonetheless, R2B can be applied to instances where FCE has already been performed. In these situations, R2B shows that the same alternative might still be chosen even with added simulated variability and adversarial inputs. Furthermore, the user is able to determine the minimum number or ratio of experts required in order to overcome any effects from an adversarial expert. However, proper use of R2B requires the user to understand handling of alternatives that end up in the same linguistic category. Users are required to understand the Max Degree of Membership and Ordered Weighted Averaging methods for breaking ties. Unfortunately, users should know that there is no standard for dealing with cases in which both methods for breaking ties conflict and lead to separate decisions.

In conclusion, FCE can provide useful information to military leaders and decision-makers requesting the optimal selection amongst several alternatives as long as its limitations are understood. Moreover, FCE offers solutions taking into consideration the imprecision and subjectivity inherent in all real situations. In order to reach this endeavor, a well-defined scenario with specified parameters are required for successful use of MCDMs such as the AHP. The bias in expert opinion found in the subjective inputs in FCE must be carefully considered. This thesis suggests implementing R2B, but this is by no means an exhaustive method for dealing with the biasness in expert opinions. R2B's capacity to evaluate variability amid inclusion of a malicious expert has implications in a combat logistics scenario. Perhaps FCE combined with other mathematical methods such as optimization can be used to support decision support processes and aid the decision-maker.

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